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# **A New Model for Estimating Bit Error Probabilities of Ring-Oscillator PUFs**

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Communication-centric Systems-on-Chip**

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# Outline

- Introduction to PUFs
- New Modeling Technique
- Empirical Results
- Conclusions



# Introduction

## Embedded security

Measurement, storage, processing,  
transmission of sensitive data

## Non-volatile memory

Secure, but expensive

## Secure key storage

Prerequisite for  
cryptography

## Technology and cost constraints

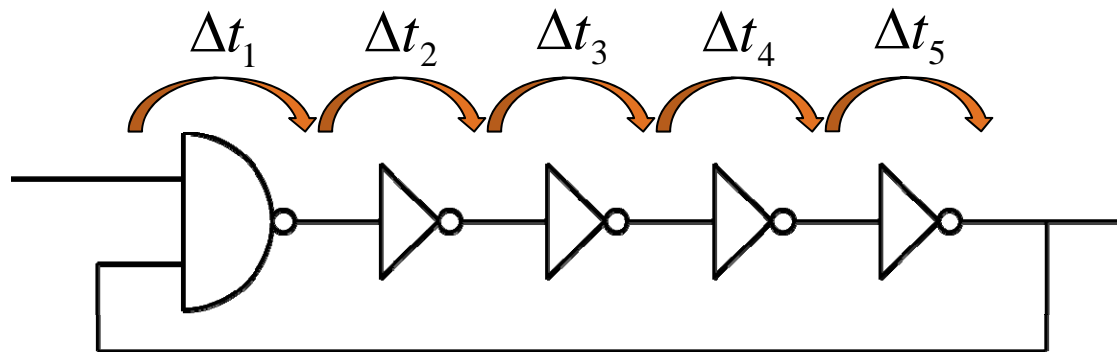
# Solution: Physical Unclonable Functions



# Physical Unclonable Functions

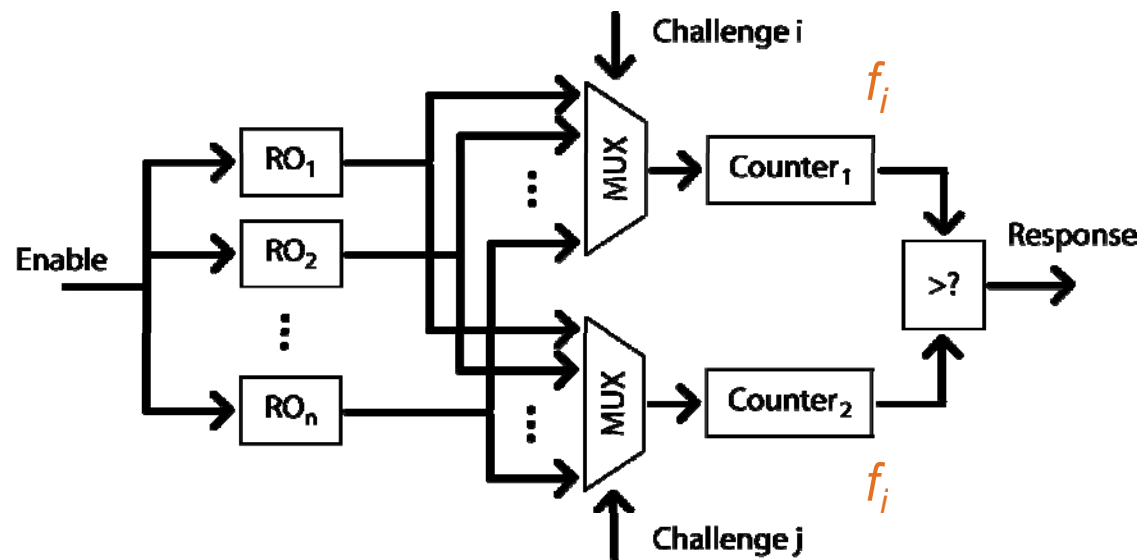
- Measurement of internal physical properties
- Randomness
- Precise PUF models necessary for error correction

# Ring Oscillators from Logic Gates



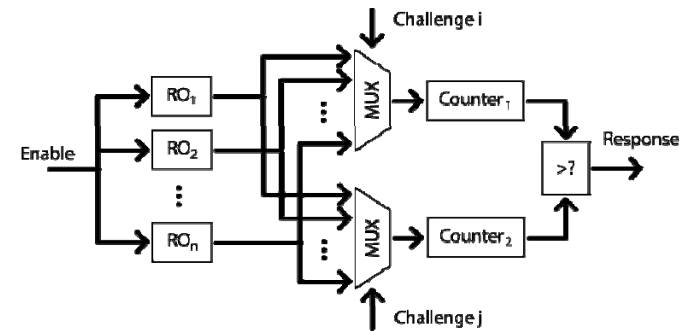
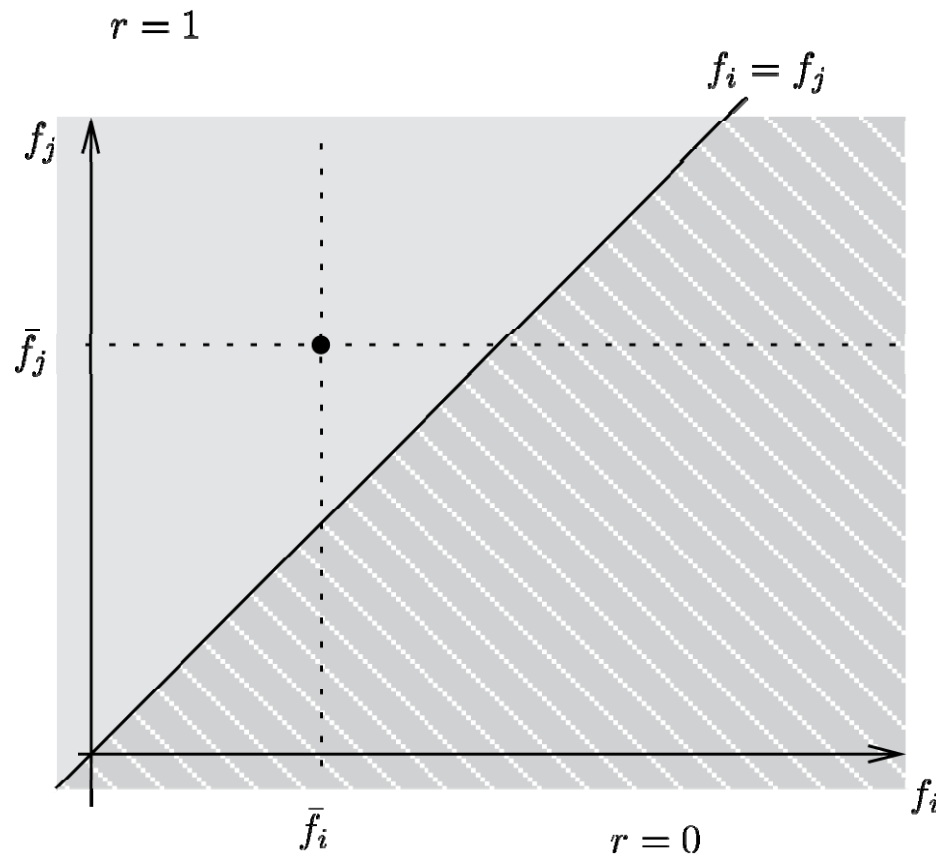
$$f = \frac{1}{\sum_{i=1}^5 \Delta t_i}$$

# Ring Oscillator PUF



Suh et al., 2007

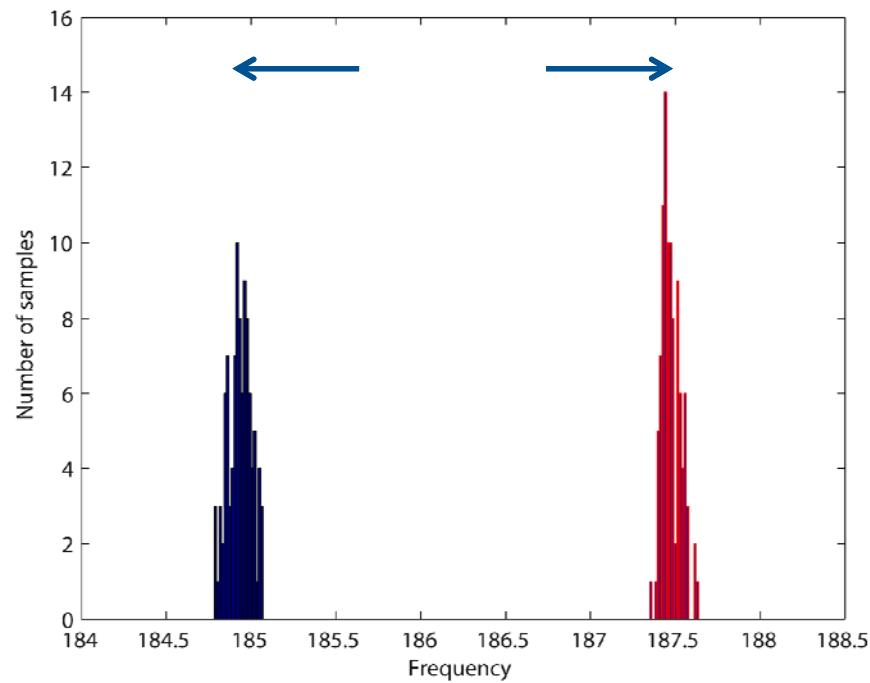
# Bit derivation



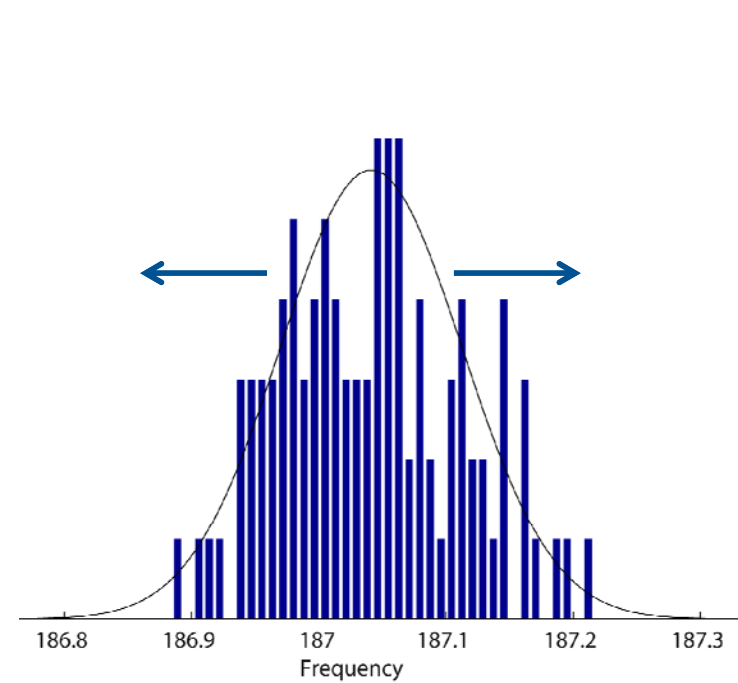


# Randomness

## Uniqueness

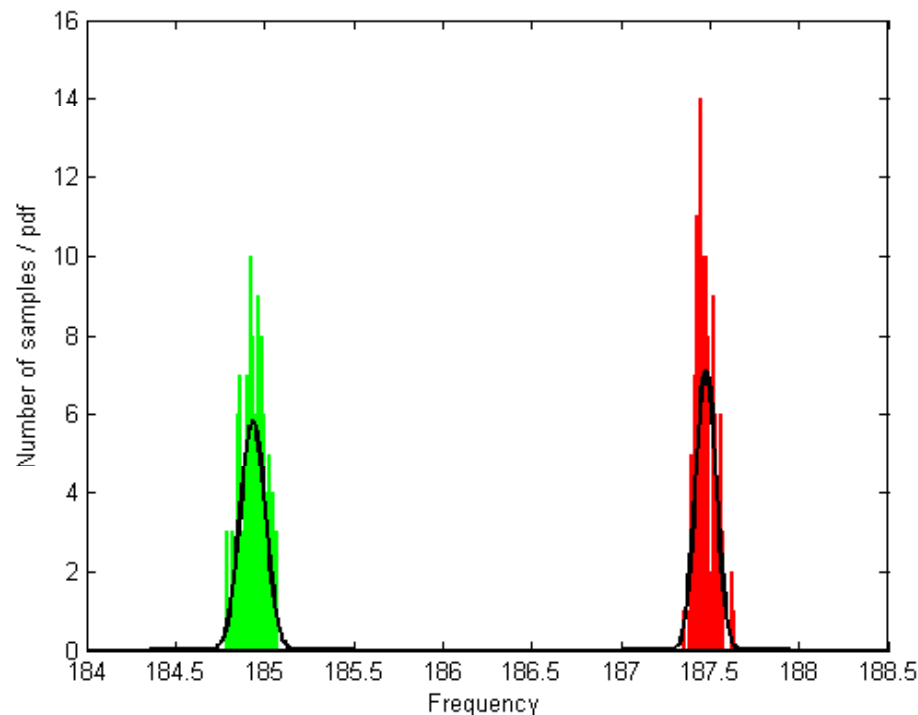


## Reliability





# State of the art



- Direct comparison of frequencies
- Problem: Precision scales linearly with the effort

e.g. Maiti et al. (HOST 2010),  
Armknecht et al. (S&P 2011)



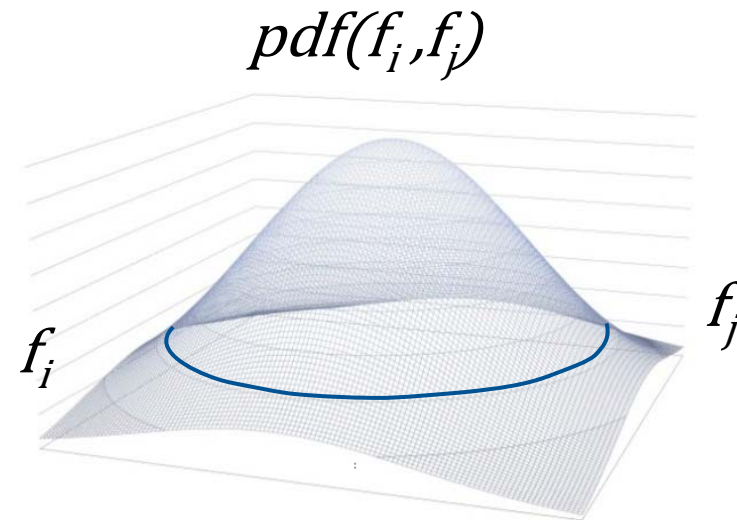
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# Multivariate Distribution

$$\bar{\mathbf{f}} = [\bar{f}_i; \bar{f}_j]^T$$

$$\mathbf{C} = \begin{bmatrix} \sigma_i^2 & \sigma_i \rho_{i,j} \sigma_j \\ \sigma_i \rho_{i,j} \sigma_j & \sigma_j^2 \end{bmatrix}$$



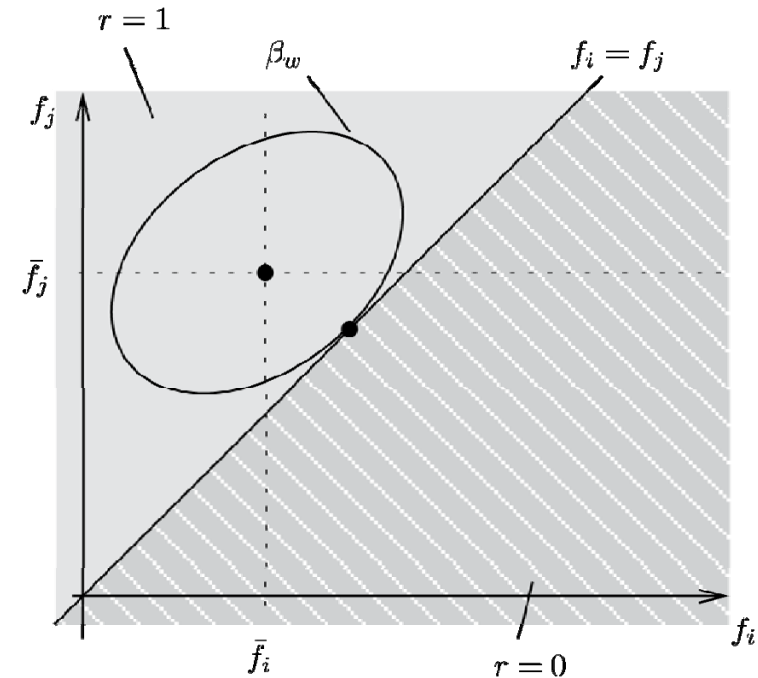
$$pdf(\mathbf{f}) = \frac{1}{2\pi \sqrt{\det(\mathbf{C})}} \exp\left(-\frac{(\mathbf{f} - \bar{\mathbf{f}})^T \mathbf{C}^{-1} (\mathbf{f} - \bar{\mathbf{f}})}{2}\right)$$

# Reliability Analysis

$$x = [1, -1] \begin{bmatrix} f_i \\ f_j \end{bmatrix} = \mathbf{m}^T \mathbf{f}$$

$$\beta = \frac{(x - \bar{x})}{\sigma_x}$$

$$\beta_w^2 = \frac{(\bar{f}_i - \bar{f}_j)^2}{\mathbf{m}^T \mathbf{C} \mathbf{m}} = \frac{(\bar{f}_i - \bar{f}_j)^2}{\sigma_i^2 - 2\sigma_i \rho_{i,j} \sigma_j + \sigma_j^2}$$



$$P(r = 1) = P(\beta < \beta_w) = \int_{-\infty}^{\beta_w} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\beta^2}{2}\right) d\beta$$



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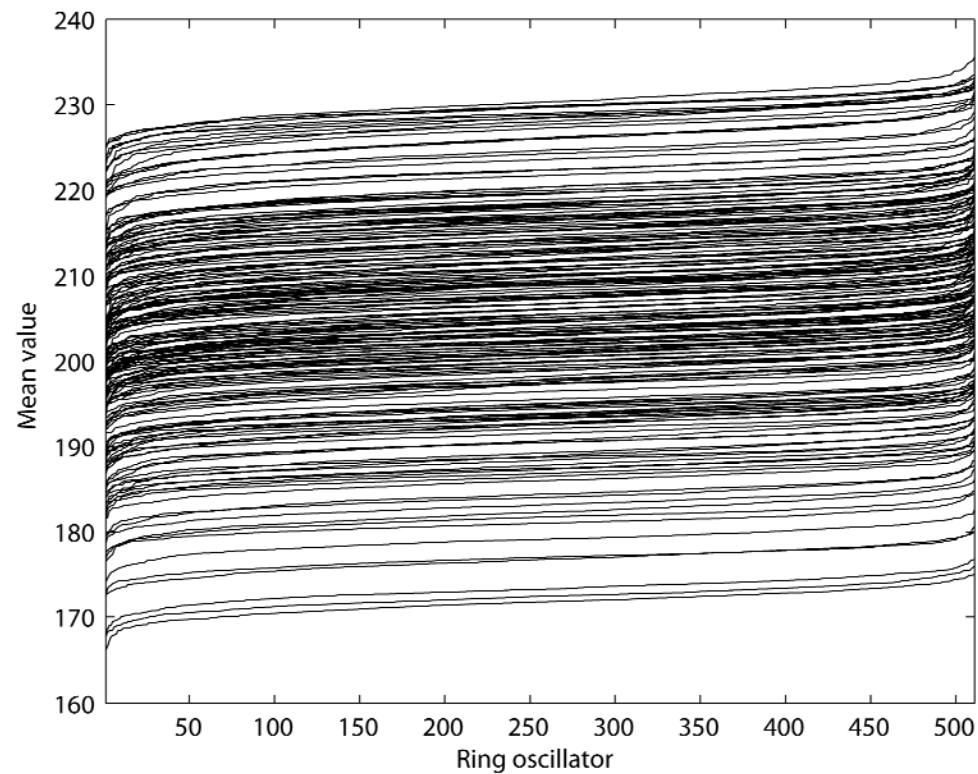
# Empirical Data

- Large scale case study at Virginia Tech in 2009
- Xilinx Spartan 3
- 193 FPGAs
- 512 ROs per FPGA
- 100 measurements per RO

<http://rijndael.ece.vt.edu/puf>

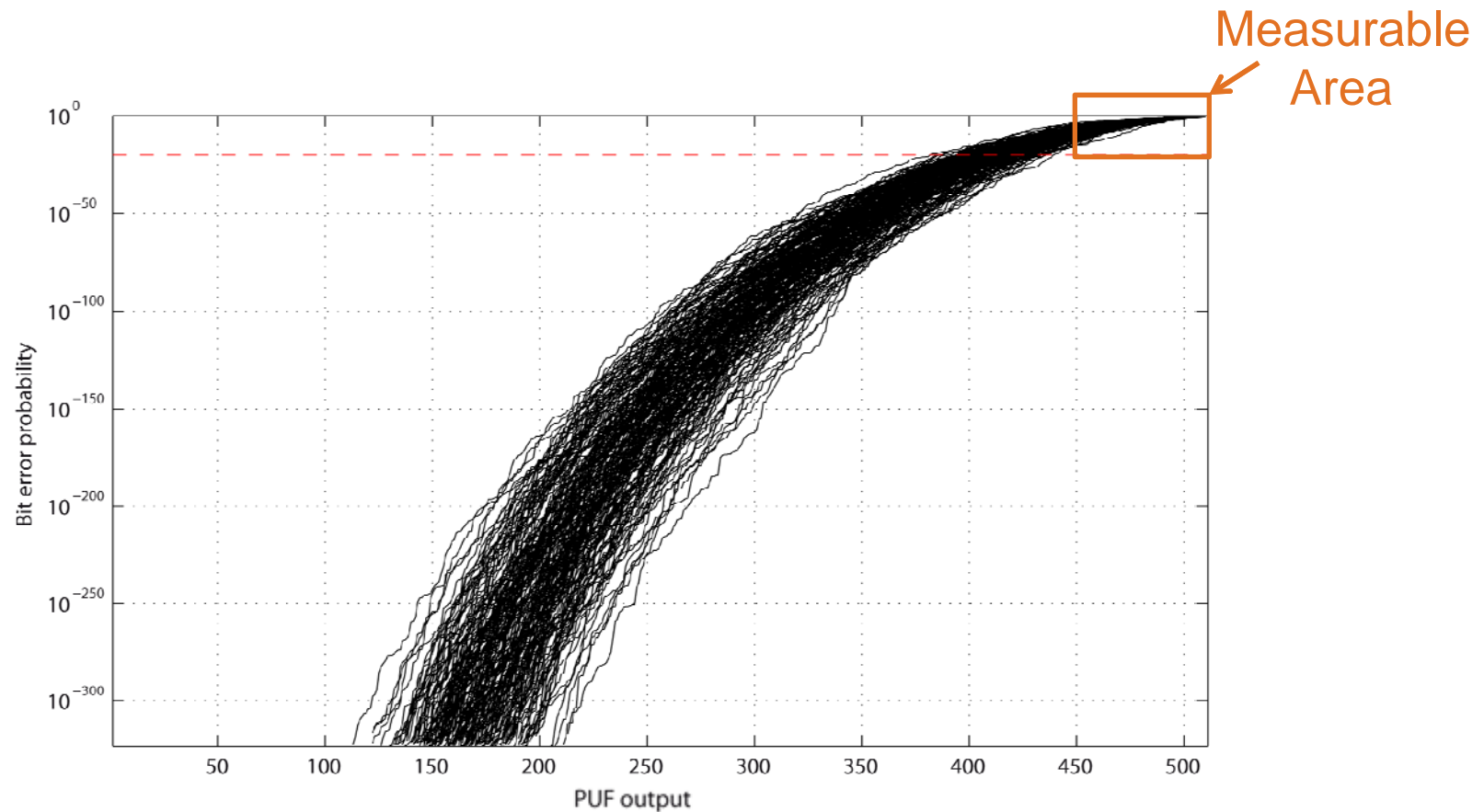


# Frequency Distribution on FPGAs



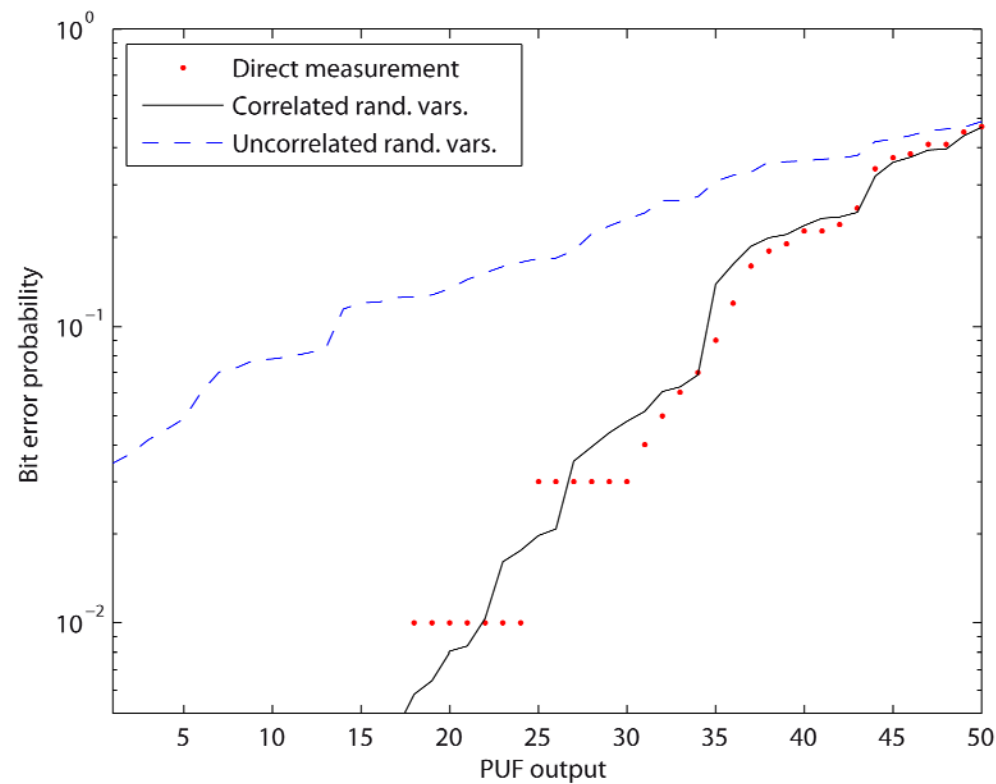


# Bit Error Probabilities with new Model





# Validation of the Model





# Conclusions

- Starting point:
  - Unknown bit error probability distribution
- New Model:
  - Correlated random variables
  - Estimation of entire bit error probability distribution
- Generalization:
  - Differential evaluation of physical measures for other PUF types (Voltage, Time, Resistance, Capacitance, ...)



# Lessons learned

- PUF measurements can be correlated, probably even under constant environmental conditions
- Theoretically founded demonstration that differential structures are useful
- RO PUF is a very reliable PUF
- New model as one step towards certification



# *Questions?*